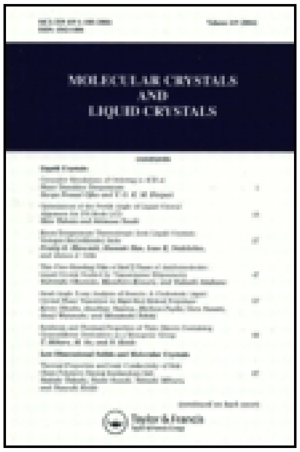

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Determination of the Director Orientation Inside a Hybrid Nematic Cell by Total Internal Reflection

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DETERMINATION OF THE DIRECTOR ORIENTATION INSIDE A HYBRID NEMATIC CELL BY TOTAL INTERNAL REFLECTION

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Abstract In this paper we report computer calculations of the reflectivity of a hybrid nematic cell near the condition of total internal reflection. The Berreman matrices method allows us to drop the approximations used in previous papers. In this way we get details about the interference pattern arising when the total internal reflection occurs inside the sample. We show how the reflectivity curve is affected by the director orientation which in a hybrid cell is essentially due to the boundary conditions. Our results suggest that measurements of this reflection pattern can give detailed information about the local director orientation.

INTRODUCTION

Total internal reflection (TIR) has been used in the past years to measure liquid crystal properties such as refractive indexes[1] and pretilt angles[2] of the boundary layers.

Recently some authors have discussed the features of the interference pattern observed in the beam reflected or transmitted by a liquid crystal near the critical angle of total reflection[3] connecting the fringes position to the distortion induced by a magnetic field in the sample[4]. The TIR method has also been used to study the Poiseuille flow in a nematic liquid crystal[5] introducing approximations able to describe that case. In this paper we use exact matrix calculations to study how the reflection pattern of a hybrid nematic liquid crystal cell depends on the director orientation inside the sample.

Here we present the results of computer calculations of the reflectivity of a hybrid nematic cell near the TIR condition. The Berreman[6] matrix method has been used to work out the reflectivity of the anisotropic inhomogeneous structure. The calculation has been performed for several different director orientation distributions inside the sample and different pretilt angles at the boundary. We find that the reflectivity pattern gives local information on the director orientation when the TIR condition is obtained inside the sample. This result suggests that measurements of this reflection pattern may allow to draw a map of the director orientation giving the tilt angle of the director in each layer.

THEORY

We consider a hybrid nematic sample with distortion in the (x,z) plane, the liquid crystal layers normal to the z axis and we define $\Phi(z)$ the tilt angle in the layer between the z axis and the local director orientation. The liquid crystal is surrounded by an isotropic and homogeneous medium with refractive index n_i, and a TM plane wave is impinging on the liquid crystal with an incidence angle Θ_i. The incidence occurs on the homeotropic side of the sample. In this geometry the matrix method introduced by Berreman can be applied to work out the reflectivity of such a sample.

The liquid crystal is divided in m layers, each layer being homogeneous but anisotropic. The transformation matrix for a layer of thickness h located at z has been obtained after a lengthy but straightforward calculation:

$$P(h,z)= \begin{vmatrix} x_3 \cos x_1 & i(a/b)x_3 \sin x_1 & 0 & 0 \\ i(b/a)x_3 \sin x_1 & x_3 \cos x_1 & 0 & 0 \\ 0 & 0 & \cos x_2 & i(1/v)\sin x_2 \\ 0 & 0 & i\, v \sin x_2 & \cos x_2 \end{vmatrix}$$

with the following definitions

$$x_1 = k_o\, a\, b\, h\,; \qquad x_2 = k_o\, v\, h$$

$$x_3 = \exp(i\, k_o\, s\, h)$$

$$a = \left[1 - \frac{n_i{}^2 \sin^2\Theta_i}{\varepsilon_o + (\varepsilon_e - \varepsilon_o) \cos^2\Phi} \right]^{1/2}$$

$$b = \left[\varepsilon_o + (\varepsilon_e - \varepsilon_o)\sin^2\Phi - \frac{((\varepsilon_e-\varepsilon_o)\sin\Phi\cos\Phi)^2}{\varepsilon_o + (\varepsilon_e-\varepsilon_o)\cos^2\Phi} \right]^{1/2}$$

$$s = -n_i \sin\Theta_i \frac{(\varepsilon_e-\varepsilon_o)\sin\Phi\cos\Phi}{\varepsilon_o + (\varepsilon_e-\varepsilon_o)\cos^2\Phi}$$

$$v = (\varepsilon_o - n_i^2 \sin^2\Theta_i)^{1/2}$$

where k_0 is the wavenumber in vacuum, ε_e and ε_o are respectively the extraordinary and ordinary complex dielectric constants of the liquid crystal.

The matrix $P(h,z)$ allows us to calculate the electromagnetic field in the usual way

$$\psi(z_2) = P(z_2 - z_1, (z_2+z_1)/2)\ \psi(z_1)$$

where

$$\psi = \begin{vmatrix} E_x \\ H_y \\ E_y \\ -H_x \end{vmatrix}$$

If a sample of thickness d is assumed to be made by m successive homogeneous anisotropic layers of thickness h the transformation matrix F of the whole sample may be computed as the product of m matrices $P(h,z_j)$, where z_j is the location of the j-th layer. The reflection and the transmission coefficients are obtained by simple algebraic relations coming from the continuity conditions for the fields. Let us consider an extraordinary wave. If I,R and T are the x components respectively of the incident, reflected and transmitted electric field amplitudes we get

$$T = f_{11} \, (I + R) + f_{12} \, r_i (I - R)$$
$$r_i \, T = f_{21} \, (I + R) + f_{22} \, r_i (I - R)$$

where f_{ij} are elements of F and $r_i = H_y/R_x = n_i/\cos \Theta_i$. Then it is straightforward to obtain the reflection coefficient:

$$R/I = \frac{r_i \, (f_{12} \, r_i - f_{22}) + (f_{11} \, r_i - f_{21})}{r_i \, (f_{12} \, r_i - f_{22}) - (f_{11} \, r_i - f_{21})}$$

In the following we are concerned with the intensity reflectivity $r = |R/I|^2$.

RESULTS AND DISCUSSION

A typical plot of the reflectivity vs. the incidence angle is shown in Fig.1. The parameters used in the calculations are: $d = 25\mu m$, $n_i = 1.9$, $\varepsilon_e = 3.01 + i0.0001$, $\varepsilon_o = 2.38 + i0.0001$, $k_o = 2\pi/0.5145\ (\mu m)^{-1}$, $m = 100$. Calculations performed with higher number of layers lead to negligible variations of the results. The imaginary part of the dielectric constant has been introduced to take into account the scattering losses in the liquid crystal. Its value is in agreement with the one usually observed in nematic liquid

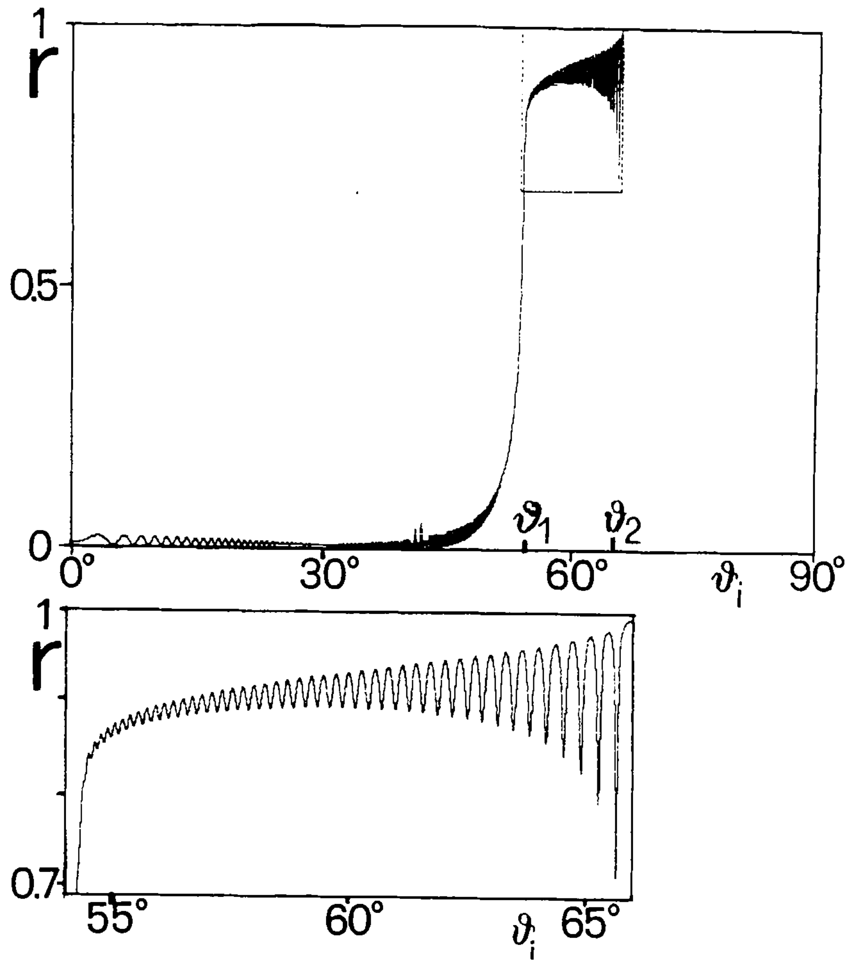

FIG.1. Typical reflectivity pattern. Inset is an enhancement of boxed area.

crystals; anyway it is not critical to get the effects reported below[7].

Three different regions are easily observed. For incidence angle $\Theta_i < \Theta_1 = 54.3°$ we have the transmission region and the well known interference fringes studied by Eidner et al[3,4]. These fringes are not resolved in this figure. They are the typical ones of a Fabry-Perot cavity. At $\Theta_i = \Theta_1$ we get the TIR condition at the second interface liquid crystal-glass, therefore we observe a sharp rise in reflectivity which anyway keeps a value lower than unity because of the scattering losses. The range $\Theta_1 < \Theta_i < \Theta_2 = 65.9°$ corresponds to the occurrence of TIR inside the liquid crystal: by varying the incidence angle, the z location where TIR occurs changes because of the different director orientations. For $\Theta_i \geq \Theta_2$ we get total reflection at the first interface glass-liquid crystal, hence the reflectivity is 1.

In this paper we are interested in the second region. In Fig.2 it has been considered in more detail. Two different director orientation $\Phi(z)$ are shown together with the correspondent reflection patterns in the range of TIR occurrence inside the liquid crystal. It is clear how a small change in the director distribution affects both number and contrast ratio of the observed fringes.

A very interesting feature is clarified in Fig.3 where the fringes correspondent to the same region of incidence angles are reported for the two director distributions $\Phi(z)$ of the inset. These "probe" distributions change abruptly in the

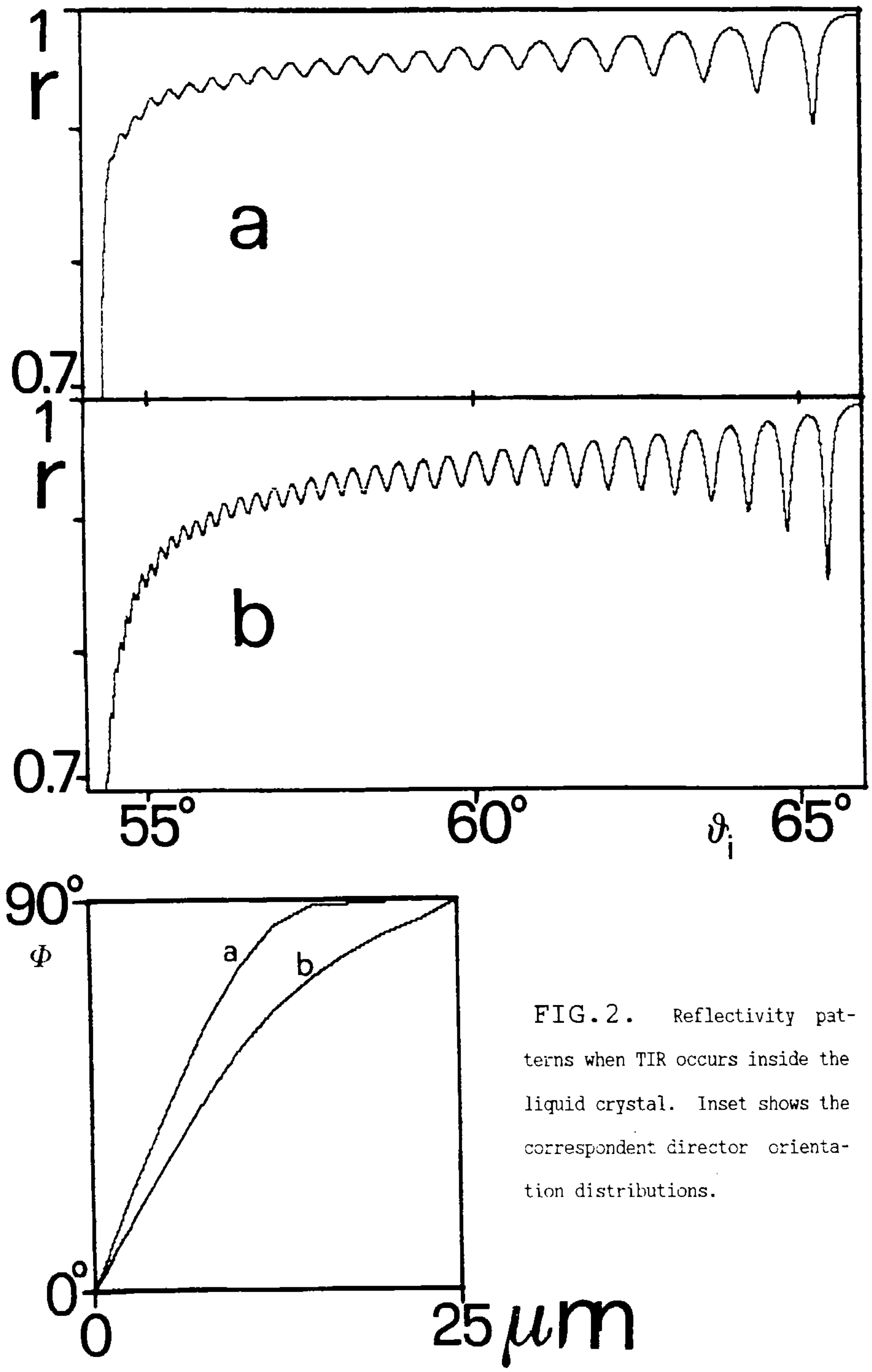

FIG. 2. Reflectivity patterns when TIR occurs inside the liquid crystal. Inset shows the correspondent director orientation distributions.

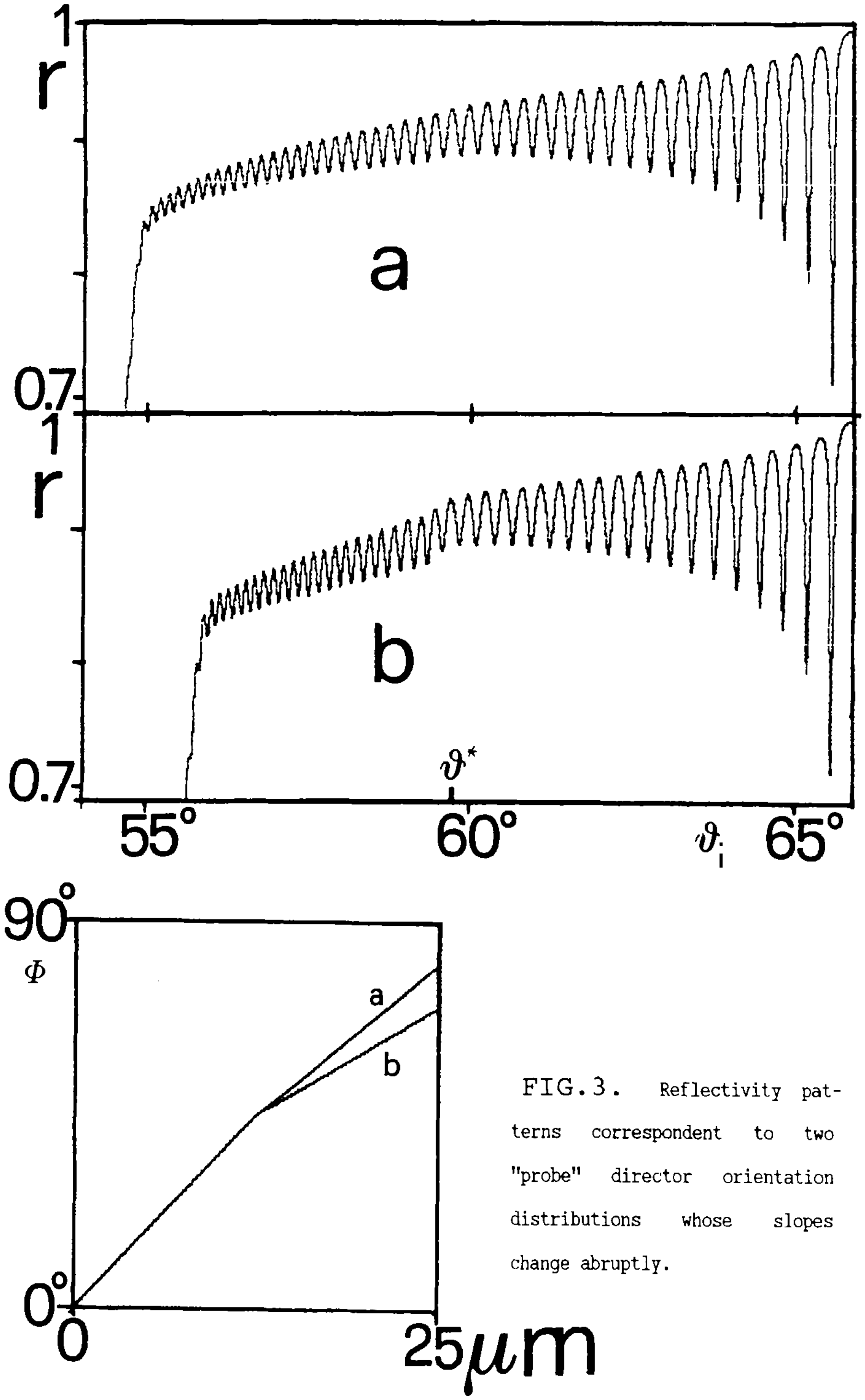

FIG.3. Reflectivity patterns correspondent to two "probe" director orientation distributions whose slopes change abruptly.

middle of the sample and after that they keep a different slope. This sudden change is observed even in the reflection curve and is more evident for case b) where the slope variation is bigger. It is interesting to notice that the fringes in the region $\Theta_i > \Theta^*$ are exactly the same for the two cases and they come from the sample region where $\Phi(z)$ is the same. In the region $\Theta_i < \Theta^*$ reflectivity patterns are clearly different as it is for $\Phi(z)$.

It must be stressed that Θ^* is the same in both cases while, of course, two different angles Θ_i correspond to the different tilt angles Φ at $d = 25\mu m$. Moreover further calculations not reported here have shown that a change in the location of the discontinuity of $\Phi(z)$ produces a correspondent change in the value of Θ^*. This example shows that the fringe pattern due to TIR inside the sample gives information about the "local" director orientation. This is the main result of our calculation. It suggests that a measurement of this reflectivity pattern can allow to draw the function $\Phi(z)$, leading also to the knowledge of the pretilt angles at the boundaries. In order to accomplish this goal one needs to work out the inverse calculation to obtain the function $\Phi(z)$ from the reflectivity pattern. Work is in progress to follow this idea.

In conclusion we have reported the calculated reflectivity pattern for a hybrid nematic cell near the total internal reflection. We have shown how the fringes, which come from TIR inside the sample, give the knowledge of the local director orientation.

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